Indian Statistical Institute

Midterm Examination 2019-2020

Analysis II, B.Math First Year

Time : 2.45 Hours Date : 24.02.2020 Maximum Marks : 100 Instructor :Jaydeb Sarkar

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Note: *(i) Answer all questions. (ii)* $R[a,b]$ = *the set of all Riemann integrable functions on* $[a,b]$.

Q1. (15 marks) It is true that $|f| \in R[a,b] \Rightarrow f \in R[a,b]$? Justify your answer.

Q2. (15 marks) Define $f : [0,1] \to \mathbb{R}$ by

$$f(x) = \begin{cases} x & \text{if } x \in \mathbb{Q} \\ 0 & \text{otherwise} \end{cases}.$$

Prove that $f \notin R[0,1]$.

Q3. (20 marks) Prove that

$$(i)\ \ln x \leq 2(\sqrt{x} - 1) \quad (\forall x \geq 1), \qquad \text{and} \qquad (ii)\ \int_0^1 x \sin^2 \frac{1}{x}\, dx \leq \frac{1}{2}.$$

Q4. (20 marks) Suppose

$$f(x) = \int_0^x (x-t) e^{\sin(\frac{t}{2020})^2}\, dt \qquad (\forall x \in \mathbb{R}).$$

Compute the value of $f''(2020)$. Justify all steps in your calculation.

Q5. (20 marks) Let (X,d) be a metric space. If $x \in X$ and $A \subseteq X$, then prove that

$$d(x, A) = d(x, \bar{A}).$$

Q6. (20 marks) Let C_0 and C_1 be disjoint closed subsets of $\mathbb{R}_u$. Prove that there exists a continuous function $f : \mathbb{R}_u \to [0,1]$ such that

$$f(x) = \begin{cases} 0 & \text{if } x \in C_0 \\ 1 & \text{if } x \in C_1. \end{cases}$$